Grade 7 Fraction Addition & Subtraction

7.N.5	
Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).	1. Model addition and subtraction of positive fractions or mixed numbers using concrete representations, and record symbolically. 2. Determine the sum of two positive fractions or mixed numbers with like denominators. 3. Determine the difference of two positive fractions or mixed numbers with like denominators. 4. Determine a common denominator for a set of positive fractions or mixed numbers. 5. Determine the sum of two positive fractions or mixed numbers with unlike denominators. 6. Determine the difference of two positive fractions or mixed numbers with unlike denominators. 7. Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator. 8. Simplify the solution to a problem involving the sum or difference of two positive fractions or mixed numbers. 9. Solve a problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

Clarification of the outcome:

- This outcome concerns understanding and being able to add and subtract fractions, and mixed numbers. Addition and subtraction can be combined because the thinking and procedures are essentially the same. There should no longer be a concern about subtraction being more difficult than addition, as there is in grade 2 and with integers.
- The lowest common denominator (LCD) approach should be the ultimate goal of instruction. The LCD is the same number as the lowest common multiple (LCM).

Required close-to-at-hand prior knowledge:

- Understand part of a whole, measurement (number line), and indicated division meanings of fraction. [This implies experience with using fraction bars, etc.]
- Understand and be able to create equivalent fractions.
- Be able to convert fractions to decimals, and to convert between fractions and mixed numbers.
- Understand primes, factors, and multiples.

SET SCENE stage

Organize students into groups. Ask them to research what occupations and/or examples from daily life that involve fractions. Some sources of information are:

http://www.xpmath.com/careers/intro.php

http://www.mathworksheetscenter.com/mathtips/fractionseveryday.html

Comments:

The purpose of the task is get students thinking about a reason for learning to work with fractions when the world around them is metric (primarily involves decimal thinking).

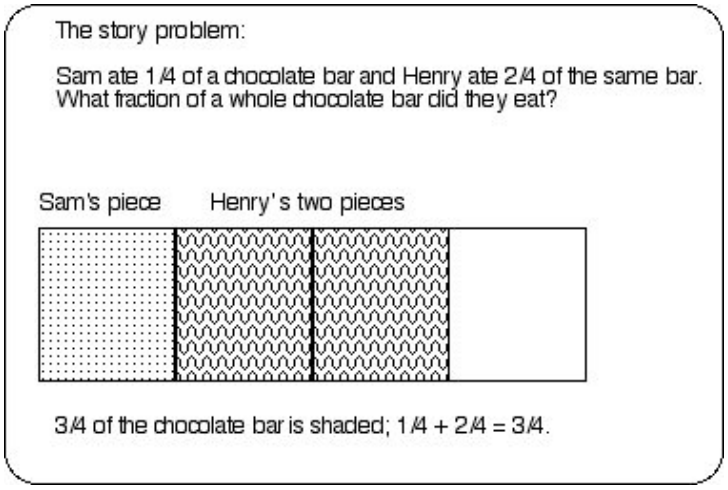
DEVELOP stage

Activity 1: Revisits SET SCENE

Ask selected groups to present their research on using fractions. Highlight any information that specifically involves fraction addition/subtraction and that involves careers in science and technology.

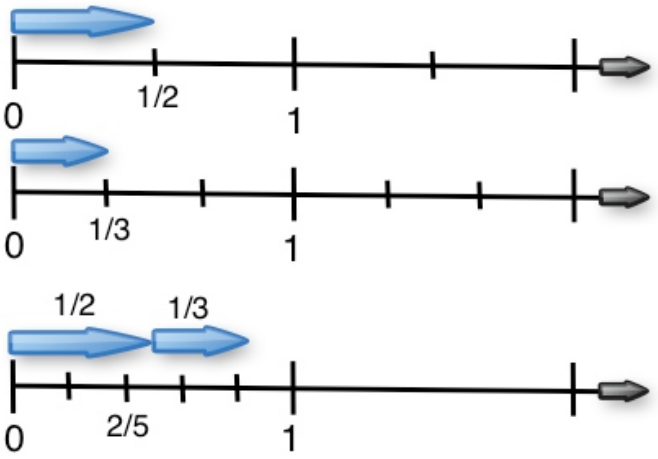
Activity 2: Addresses achievement indicators 1, 2, 3, 8, and 9.

- Have students create one fraction addition and one subtraction story problem involving like denominators. Have them solve the problems using circles, fraction bars, or number lines (see example). Then have them determine a symbolic method for adding and subtracting fractions having like denominators (e.g. 2/8 + 3/8 = (2 + 3)/8 = 5/8). Discuss their methods.
- Have students test their method for adding or subtracting fractions having like denominators by presenting them with one fraction addition and one fraction subtraction question (e.g. 3/6 + 1/6; 4/5 - 3/5). Have students use their method to obtain the answers. Have students use diagrams and/or concrete materials to confirm that their method results in the same answer as that obtained by using diagrams or concrete materials.



Activity 3: Addresses achievement indicators 1 and 5.

- Pose the question: **Why is the answer to 1/2 + 1/3 not 2/5?** Discuss student responses.
- Use a number line model to illustrate why the answer is almost 1 (and thus greater than 2/5). Ensure that 2/5 is marked on the number line. Discuss why the model shows that 1/2 + 1/3 is not 2/5.



Activity 4: Addresses achievement indicators 1, 2, 3, and 8.

- Pose the problem: "*Find five fraction pairs having the same denominators whose sum equals 1.*" Discuss students' solutions ·(e.g. 2/3 + 1/3 = 1; 5/10 + 5/10 = 1). Use diagrams/concrete materials as needed.
- Present the following five tasks: (1) 3/5 + ? = 7/5, (2) 6/8 - ? = 2/8, (3) ? + 3/10 = 1 1/10, (4) ? + ?? = 5/6, and (5) ? - 3/12 = 9/12. Ask students to figure out the answers. Discuss their solutions. Use diagrams/concrete materials as needed.

Activity 5: Addresses achievement indicators 2, 3, and 8.

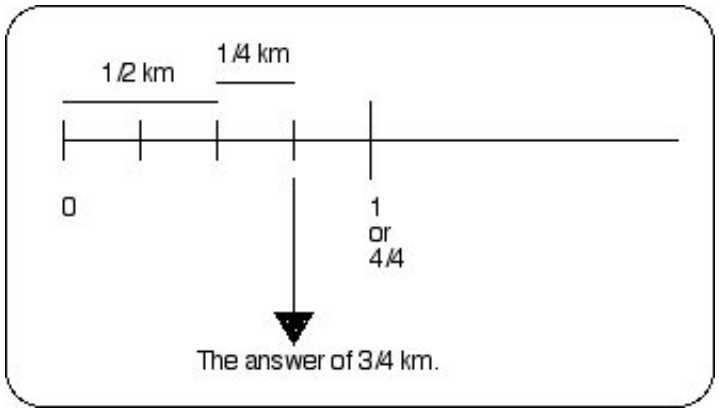
- Present a fraction addition and a fraction subtraction question (like denominators situation), where the fractions are not repeating decimals (e.g. 3/5 + 1/5; 7/10 - 2/10). Ask students to use their method to obtain the answers. Ask students to confirm the answers by converting the fractions to decimals and checking the decimal addition (for example: 3/5 + 1/5 = 4/5 converts to .6 + .2 = .8, and .8 is the correct answer to the decimal addition).

Activity 6: Addresses achievement indicators 1, 2, 3, 8, and 9 (mixed numbers).

- Have students solve the problem: "*A consulting company made the following estimates for repairing a building gutted by fire: 3/8 month for cleaning out the debris, 1/8 month for planning, and 3 1/8 months for rebuilding. How many months does the company think are needed for repairing the building?*" Discuss students' solutions. Use diagrams/ concrete materials as needed to confirm the answer.
- Present decontextualized addition and subtraction tasks involving mixed numbers, where the fractions have like denominators (e.g. 1 3/4 + 2/4 + 1/4; 2 3/8 - 1 1/8). Ensure that students understand how to add and subtract mixed numbers, when the fractions have like denominators.

Activity 7: Addresses achievement indicators 1, 4, 5, 6, 8, and 9.

- Pose the problem: "*Mary walked 1/2 kilometre on Monday and 1/4 of a kilometre on Tuesday. What fraction of a kilometre did Mary walk altogether?*" Ask students to represent the problem on a number line marked off into quarters and then obtain an answer using the number line. Discuss their solutions.



- Pose the problem: "*Mary walked 3/8 kilometre on Monday and 1/4 of a kilometre on Tuesday. What fraction of a kilometre did Mary walk altogether?*" Ask students to represent the problem on a number line marked off into eighths and then obtain an answer using the number line. Discuss their solutions.
- Ask students to propose a symbolic method for adding fractions having unlike denominators, based on their experiences with the previous two problems (e.g. Need to make all denominators the same, then add; for example, 3/8 + 1/4 = 3/8 + 2/8 = 5/8). Have students confirm their method by creating addition tasks involving unlike denominators (e.g. 1/5 + 7/10), obtaining an answer by using the method, and then checking the answer by using concrete materials/diagrams.
- Ask students if the symbolic method would also work for subtracting fractions with unlike denominators. Ask them to provide examples. Discuss them.

Activity 8: Addresses achievement indicators 5, 6, and 8.

- Present a fraction addition and a fraction subtraction question (unlike denominators situation), where the fractions are not repeating decimals (e.g. 1/5 + 3/10; 3/4 - 1/8). Ask students to use their method to obtain the answers. Ask students to confirm the answers by converting the fractions to decimals and checking the decimal addition (for example: 1/5 + 3/10 = 5/10 converts to .2 + .3 = .5, and .5 is the correct answer to the decimal addition).

Activity 9: Addresses achievement indicators 5, 6, 8, and 9.

- Ask students to solve two simple fraction addition problems involving 1/2's and 1/4's (e.g. *I ate 1/4 of a large pizza. My dad ate 1/2 of it. What fraction of the pizza did we eat altogether?*). Have students use a symbolic method (from activity #6) to do the addition and confirm the answer by using concrete materials/diagrams.
- Ask students to solve two simple fraction subtraction problems involving 1/2's and 1/4's (e.g. *Mom gave me 1/2 of a pizza for lunch. I ate 1/4 of it. What fraction of the pizza is left?*). Have students use a symbolic method to do the subtraction and confirm the answer by using concrete materials/diagrams.
- Have students solve a floor construction problem: "*A subfloor for ceramic tiles is made by nailing two layers of plywood to the floor joists. The first layer of plywood is a 1/2 inch thick and the second layer is 3/8 inches thick. How thick is the subfloor?*" Have students use a symbolic method to do the addition and confirm the answer by using a number line.
- Have students solve a floor renovation problem: "*A floor 3/4 inches thick is made up of two layers. The top layer of 1/2 inch thick oak is removed because it is worn out. What is the thickness of the remaining floor?*" Have students use a symbolic method to do the subtraction and confirm the answer by using a number line.

Activity 10: Addresses achievement indicators 5, 6, 8, and 9 (mixed numbers).

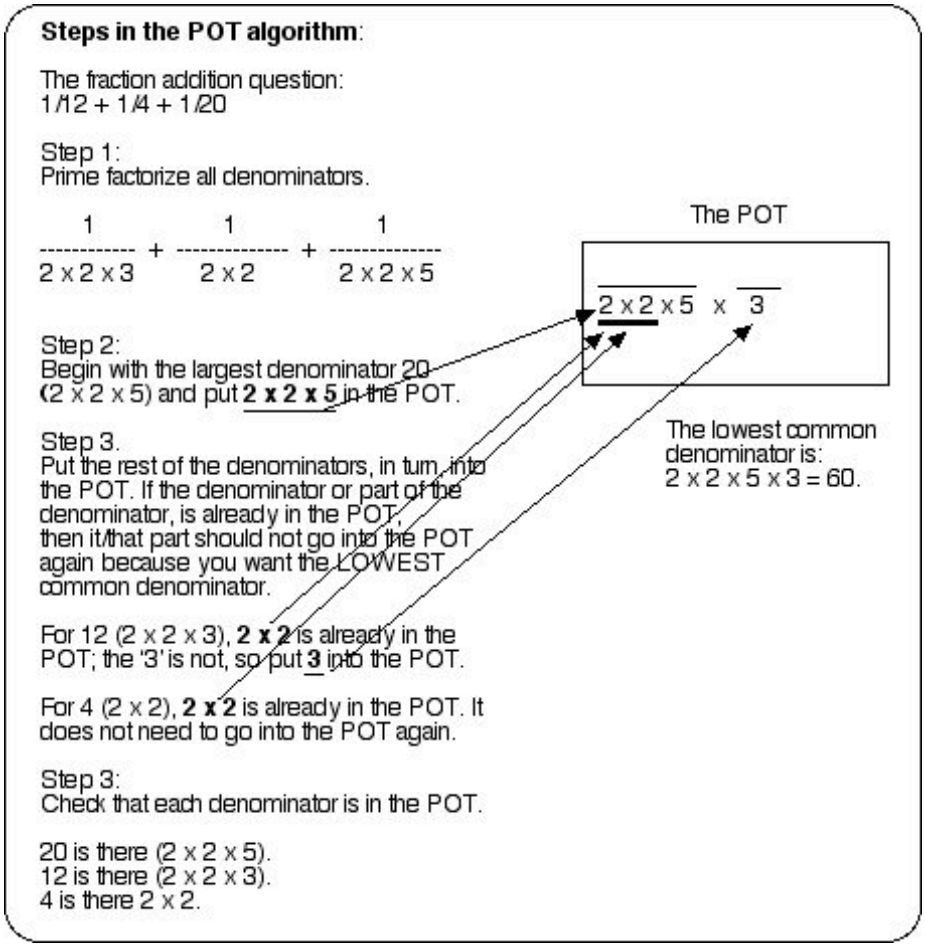
- Have students solve the problem: "*A consulting company made the following estimates for repairing a building gutted by fire:1 3/8 months for cleaning out the debris, 1/4 month for planning, and 3 1/2 months for rebuilding. How many months does the company think are needed for repairing the building?*" Discuss their solutions. Use diagrams/concrete materials as needed to confirm the answer.
- Present a variety of decontextualized addition and subtraction tasks involving mixed numbers, where the fractions have the unlike denominators (e.g. 1 3/8 + 2/5 + 1/4 = ?; 2 5/12 - 1 1/3 = ?). Some of the subtraction questions should involve a need to trade a '1' for the equivalent fraction name (e.g. 2 1/3 - 1 4/5). Ensure that students understand how to add and subtract mixed numbers and fractions, when the fractions have unlike denominators.

Activity 11: Addresses achievement indicators 5, 6, 8, and 9 (mixed numbers).

- Ask the groups from SET SCENE stage to each create one fraction addition or subtraction word problem that is related to the occupation of real life example they did their presentation on (see activity 1). [Teacher must ensure that the problem is appropriate.]
- Groups share and solve each others word problems. Discuss solutions.

Activity 12: Addresses achievement indicators 5, 7, and 8.

- Present students with an addition task involving three unit fractions, where there is significant complexity of overlap of the denominators (e.g. 1/12 + 1/4 + 1/20). Ask students to find a common denominator. Assist as required. Ask students to find the lowest common denominator (expect 60). Assist as required. [DO NOT actually complete the addition.] Discuss students' methods for finding the lowest common denominator (some students will use methods that involve dividing entire denominators). Have students determine if the lowest common denominator is the same as the lowest common multiple of the three denominators (could skip count each denominator, stopping when they find the first skip counting number that is common to all three skip counting lists: 12, 24, 48, **60**; 4, 8, 12, 16, 20, . . . 56, **60**; 20, 40, **60**). Suggest that there is a nice way of finding the lowest common denominator that makes use of prime factors.
- Revisit prime factorization. Ensure that students can determine the prime factorization of numbers such as 12, 20, 36, 48, etc. (e.g. the prime factorization of 12 is 2 x 2 x 3).
- Revisit the three-fraction addition question. Have students determine the prime factorization of each denominator. Ask students how they could use the prime factorization to find the lowest common denominator. Discuss their methods. Ensure that students realize that by putting prime factors into a POT, where you DO NOT repeat factors that are already in the POT, you end up with the lowest common denominator in the POT.



Complete the POT algorithm for the three-fraction addition question by writing equivalent fractions as required and then adding (see below).

Completing the POT algorithm:

The fraction addition question:
1/12 + 1/4 + 1/20

$$\frac{1}{2 \times 2 \times 3} + \frac{1}{2 \times 2} + \frac{1}{2 \times 2 \times 5}$$

The LCD

$2 \times 2 \times 5 \times 3$

$$\frac{1 \times \mathbf{5}}{2 \times 2 \times 3 \times \mathbf{5}} + \frac{1 \times \mathbf{3 \times 5}}{2 \times 2 \times \mathbf{3 \times 5}} + \frac{1 \times \mathbf{3}}{2 \times 2 \times 5 \times \mathbf{3}}$$

Explanation:

The simplest way to make equivalent fractions, each having 2 x 2 x 5 x 3 as the denominator, is to think in terms of "what is missing".

For 1/12, x 5 was used because it is missing from 2 x 2 x 3.

For 1/4, x 3 x 5 was used because it is missing from 2 x 2.

For 1/20, x 3 was used because it is missing from 2 x 2 x 5.

Writing the equivalent fractions and adding:

$$\frac{5}{60} + \frac{15}{60} + \frac{3}{60} = \frac{23}{60}$$

Provide students with decontextualized practice of fraction addition and subtraction involving at least three fractions, where the POT algorithm is best suited for determining the LCD (e.g. 3/4 + 5/24 + 2/15). Assist as required.

Activity 13: Assessment of teaching.

- Provide students with a two fraction addition tasks (one like and one unlike denominators) and a subtraction task (unlike denominators). Have them to do the tasks using a symbolic method and then confirm the answers to the addition tasks by using models (number line or fraction bar).

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

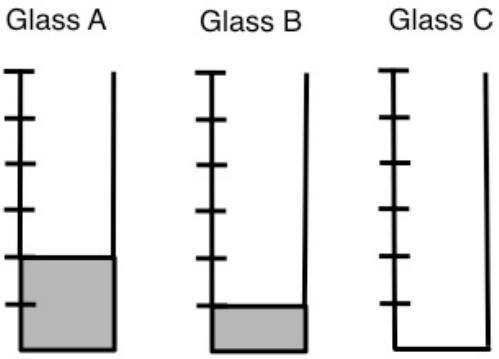
The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

Imagine pouring the liquid from glasses A and B into glass C. Shade in the amount that would be Glass C. Then complete the addition sentence.

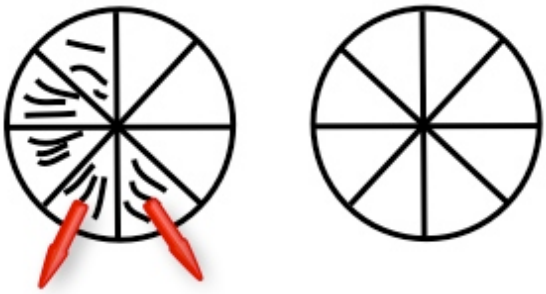
__ /6 + __ /6 = _____



Question 2.

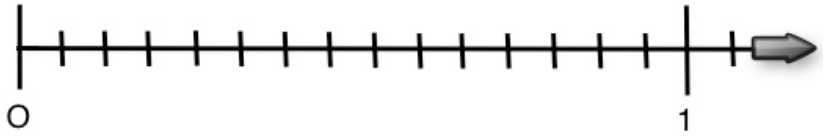
Shade in the unshaded circle the fraction of pie left if you took away the fraction shown in the shaded circle. Then complete the subtraction sentence.

__ /8 - __ /8 = _____



Question 3.

a) Show on the number line why 3/10 + 1/5 does not equal 4/15.



b) Explain in words why the number line shows that 3/10 + 1/5 does not equal 4/15.

Question 4.

DO NOT do the addition or subtraction. Make skip-counting lists to figure out the common denominator that is needed to add or subtract the following fractions.

a) 1/3 + 7/12

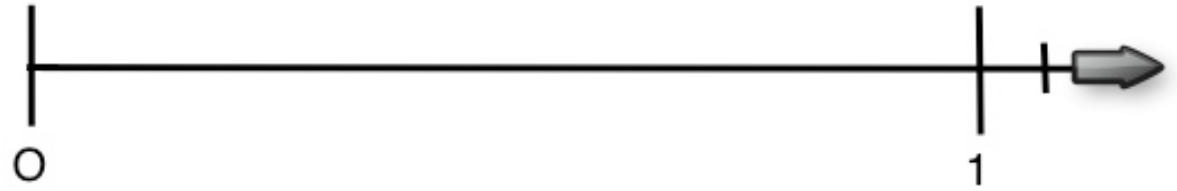
b) 3/10 - 1/5

c) 7/12 - 3/8

d) 1/2 + 1/3 + 1/4

Question 5.

Use a number line to obtain the answer to 1/2 + 3/8. Explain the thinking involved.



Question 6.

Obtain the answer to each addition or subtraction question. Show your work and explain how you figured out what the lowest common denominator is.

a) 1/2 + 1/3

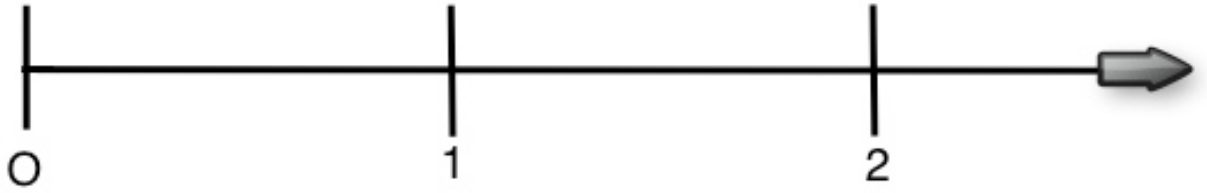
b) 1/4 + 3/5

c) 7/8 - 2/3

d) 1/2 + 1/4 + 1/8

Question 7.

Use a number line to obtain the answer to 1/2 + 1 3/4. Explain the thinking involved.



Question 8.

Obtain the answer to each addition or subtraction question. Show your work and explain how you figured out what the lowest common denominator is.

a) 1 3/7 + 6/7

b) 1 3/10 + 2 9/10

c) 3/4 + 7/8

d) 1 2/3 + 4 2/5

e) 1 7/8 - 5/8

f) 2 3/5 - 1 1/5

g) 3 1/2 - 1 2/3

MAINTAIN stage

Mini-task example

Every so often:

- Present a two fraction addition/subtraction tasks (unlike denominators), where one of the tasks involves mixed numbers.

Rich-task example

Have students solve the following problem symbolically and check the solution by using concrete materials/diagrams.

John is designing a picture page for the yearbook. He already has four pictures to use. One of them takes up 3/16 of the space on the page; the second takes up 1/4 of the space, the third takes up 1/24 of the space, and the fourth takes up 1/8 of the space. He wants to put in a fifth picture that requires 1/5 of the space. Will there be enough space to place the fifth picture?

Comments

This is a rich-task because it is a complex problem involving fraction addition and subtraction.